ALL SAINTS' COLLEGE

Ewing Avenue, Bull Creek, Western Australia

Year 12 Physics 3AB   Motion Test 2   March 2013

Student Name: Solutions

Time allowed: 45 minutes
Total marks available: 45
Show calculation answers to 3 significant figures

1. The **Mir Space Station** was a Soviet research facility in a low Earth orbit that operated until 23rd March 2001: it had an altitude of 350 km.

Calculate the gravitational field strength of the Earth at this altitude. (3)

$r = r_e + alt$

$r = 6.38 \times 10^6 + 350 \times 10^3$

$r = 6.73 \times 10^6$ m

$g = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6.73 \times 10^6)^2}$

$g = 8.79$ N kg^{-1}

2. Explain clearly why the astronauts on board the Mir space station have no apparent weight. (2)

Both the astronaut and the satellite are in circular motion such that centripetal force is provided only by weight (effectively in freefall)

$\therefore$ no normal reaction $\left(\frac{v^2}{r} = g\right)$

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3. Venus is the second planet from the Sun. The mass of Venus is 4.83×10^{24} kg. The distance between Venus and the Sun is 1.08×10^{8} km. The radius of Venus is 6.31×10^{6} m.

a) Calculate the gravitational force of the Sun acting on Venus. (3)

$M_{sun} = 1.99 \times 10^{30}$ kg $\quad m_{venus} = 4.83 \times 10^{24}$ kg $\quad r = 1.08 \times 10^{11}$ m

$$F = G\frac{m_1 m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 4.83 \times 10^{24} \times 1.99 \times 10^{30}}{(1.08 \times 10^{11})^2}$$

$F = 5.50 \times 10^{22}$ N towards sun

b) Assuming Venus has a circular orbit calculate its orbital speed around the sun. (3)

$$\frac{v^2}{r} = G\frac{m}{r^2}$$

$$V = \sqrt{\frac{G M_{host}}{r}}$$

$M_{host\ (sun)} = 1.99 \times 10^{30}$ Kg

$r = 1.08 \times 10^{11}$ m

$$V = \sqrt{\frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30}}{1.08 \times 10^{11}}}$$

$V = 3.51 \times 10^{4}$ m s^{-1}

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4. The Millennium Falcon has been placed in a stable orbit around the planet Tatooine. The distance between the centre of Tatooine and the centre of mass of the Millennium Falcon is 10,700 km with an orbital period of 3 hours and 22 minutes. The mass of the Millennium Falcon is 7,170 kg.

The Millennium Falcon placed in a stable orbit around the planet Tatooine

a) Calculate the mass of Tatooine from this data. (4)

$m_{tat} = ?$ $T = 3\times 60\times 60 + 22\times 60$ $r = 10,700,000$ m

$T = 12,120$ s

$\frac{mv^2}{r} = G\frac{m_1 m_2}{r^2}$ $\therefore r^3 = \frac{G m_{host}}{4\pi^2}\times T^2$

alt

(1) $V = \frac{2\pi r}{T}$

(2) $\frac{mv^2}{r} = \frac{Gm_1 m_2}{r^2}$

$\frac{v^2}{r} = \frac{Gm_2}{r}$

$(10,700,000)^3 = \frac{6.67\times 10^{-11}\times m_{tat}}{4.\pi^2}.(12120)^2$

$m_{tatooine} = 4.94\times 10^{24}$ kg

b) Calculate the acceleration of the Millennium Falcon relative to Tatooine using the above data. (4)

or $V = \frac{2\pi r}{T} = \rightarrow$

$a = \frac{V^2}{r} = \frac{4\pi^2 r}{T^2}$

identifies variables

$a = \frac{F}{m}$ $\left(F = \frac{Gm_1 m_2}{r^2}\right)$

$a = \frac{4\times\pi\times 10,700,000}{(12,120)^2}$

$g = \frac{GM}{r^2} = a$

$a = 2.88$ m s^{-2} to'ord centre Tatooine

5. Satellites, which are in orbits around planets such that they remain over one location, as observed from the planet's surface, are said to be "synchronous". Describe **3 features** of a "synchronous" orbit.

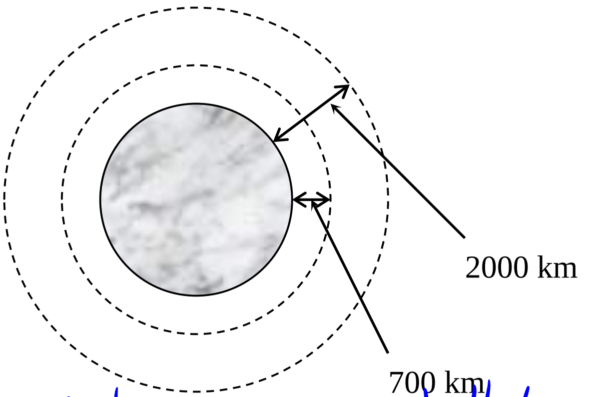
(3)

period of rotation matches the 'day' of planet
(if student has noted sidereal day, all well and good)
fixed altitude determined by $r^3 = \frac{G M_{planet}}{4 \pi^2} \cdot T^2$
equatorial orbit
circular orbit

any 3

6. A communications company operates several satellites around the Earth. They must move one of the satellites from an altitude of 700 km to an altitude of 2000 km The CEO insists that the satellite must maintain the same orbital period at this new altitude. Explain whether or not this is possible.

(3)



Condition for satellite
field strength must match
centripetal acceleration at location of satellite

From this we can derive the equation for Kepler's 3rd law $r^3 = \frac{GM}{4\pi^2} T^2$

This shows that for any given radius there is only one possible period. Therefore not possible.

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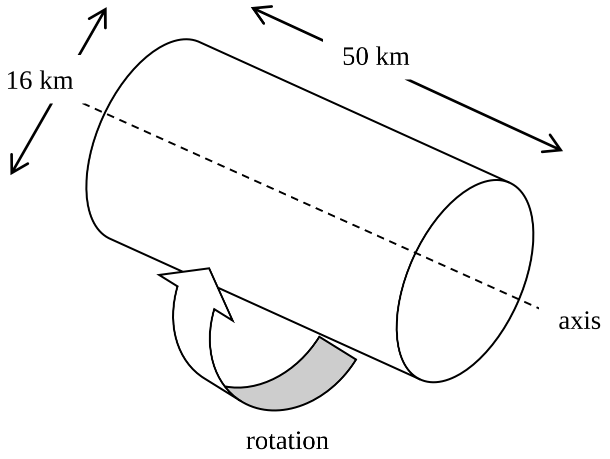
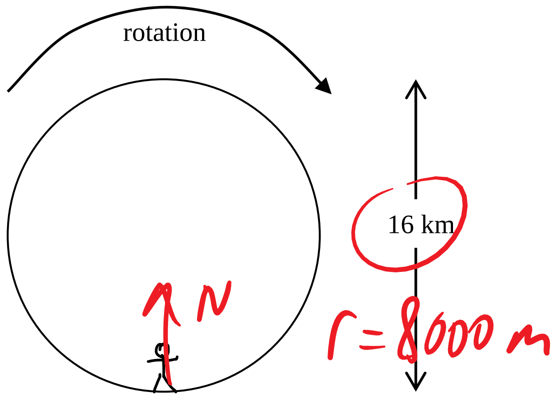
7. Rendezvous with Rama is a novel by Arthur C. Clarke published in 1972. The story involves a 16 km wide and 50 km long cylindrical alien starship that enters the solar system. The cylinder has a period of rotation of 4 minutes about an axis along the centre of its length. The cylinder is sealed and has an atmosphere within its hollow interior.





End view of cylinder – person standing on floor inside the cylinder (not to scale)

a) Calculate the apparent weight of a 70 kg person placed firmly on the floor within the cylinder as it rotates. (You can disregard the gravitational fields of any objects) (4)

$T = 4 \times 60 = 240$ seconds $\qquad r = 8{,}000$ m

$F_{centripetal}$ provided by Normal Reaction

$$\frac{mv^2}{r} = N \qquad V = \frac{2\pi r}{T} \qquad \therefore N = \frac{m\,4.\pi^2.r}{T^2}$$

$$N = \frac{70 \times 4.\pi^2.8000}{240^2} = 384 \text{ newtons}$$

b) Calculate the period of rotation required to create a sensation of apparent weight the same as standing on the surface of Earth. (3)

$a_c = 9.80$ m/s^2

$$a_c = \frac{4\pi^2 r}{T^2}$$

$$T = \sqrt{\frac{4.\pi^2 r}{a_c}}$$

$$T = \sqrt{\frac{4.\pi^2.8000}{9.8}}$$

$T = 180$ second

(3 mins)

alt $\frac{v^2}{r} = 9.80 \rightarrow v = 280$ m/s $\qquad T = \frac{2\pi r}{v} = 180$ s

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8. A team of astronauts is investigating the relationship between the gravitational field strength of Mars and the altitude above the surface of Mars. A gravitometer inside their space ship measures the field strength at different altitudes. Mars has a radius of 3.43×10^6 m. The results are recorded in the table below.

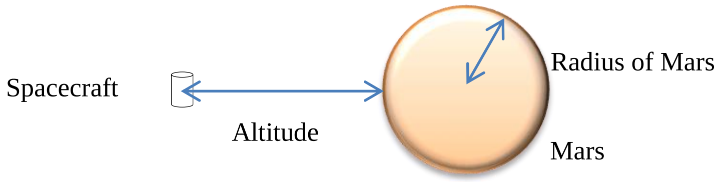

Table of results:

Altitude (m)	Radius of separation – r (m)	$1/r^2$ (m^{-2})	Field strength ($N\ kg^{-1}$)
0.00	3.43×10^6	8.50×10^{-14}	3.61 ± 0.4
3.30×10^5	**3.76×10^6**	**7.07×10^{-14}**	2.85 ± 0.3
7.80×10^5	**4.21×10^6**	**5.64×10^{-14}**	2.30 ± 0.2
1.43×10^6	**4.86×10^6**	**4.23×10^{-14}**	1.95 ± 0.2
2.52×10^6	**5.95×10^6**	**2.82×10^{-14}**	1.10 ± 0.1
4.98×10^6	**8.41×10^6**	**1.41×10^{-14}**	0.60 ± 0.06

The gravitometer gave values with an uncertainty of ± 10% so the astronauts expressed their values of field strength with this degree of uncertainty in the results table.

The astronauts were confident that the altitude measurements were highly accurate and decided to express these without any uncertainty.

The relationship between field strength and radius of separation is given by the equation:

gradient

y $g = \frac{GM}{r^2}$ x   this equation follows the format y = m.x + c $\frac{1}{r^2} = x$

g = gravitational field strength ($N\ kg^{-1}$)

G = universal gravitational constant

M = mass of object responsible for field (kg)

r = the radius of separation (m)

Answer the following questions

a. Complete the table by filling in the missing values and also fill in the values of uncertainty that are missing in the final column.

(3)

Mark for each correct column ✓ ✓ ✓

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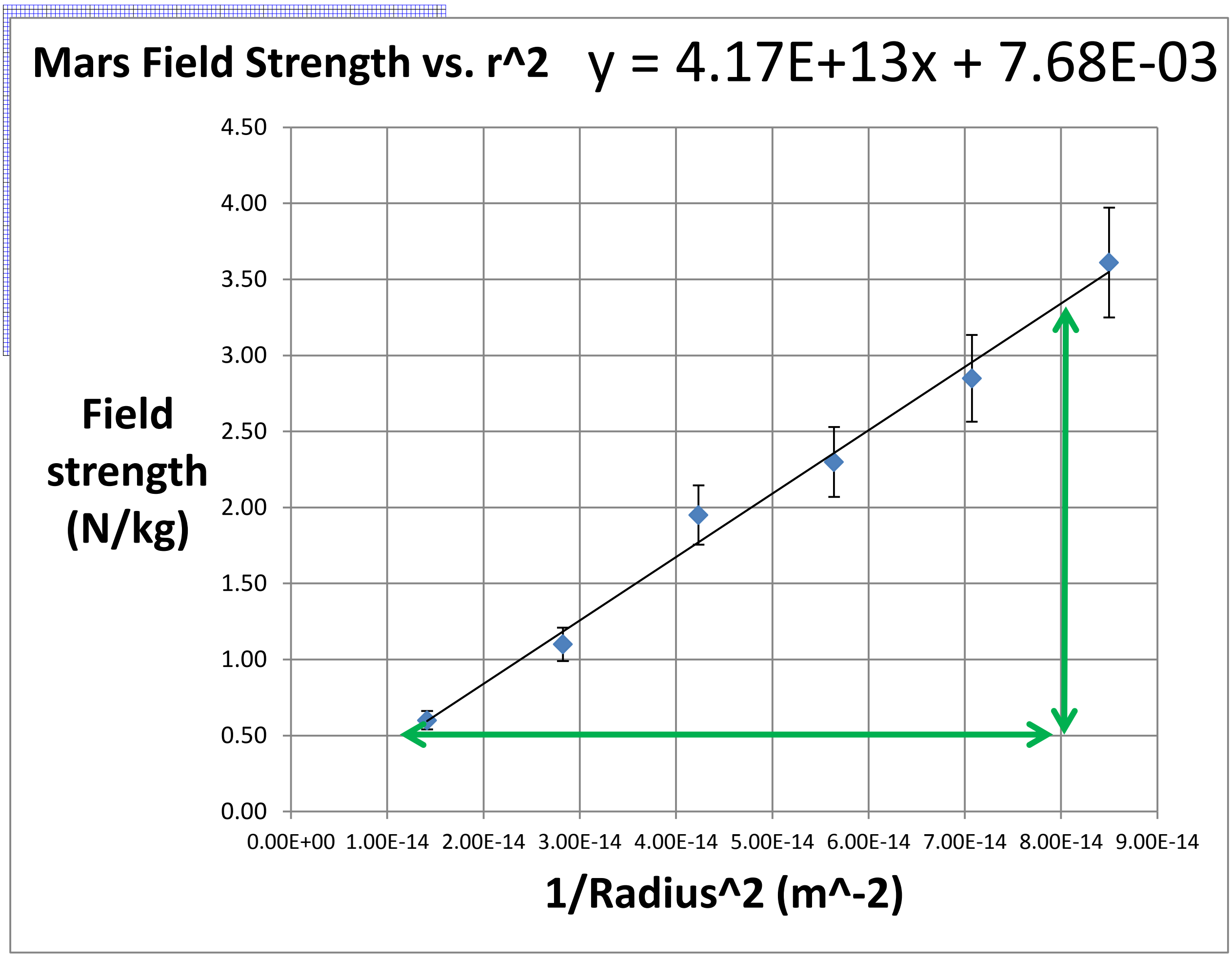
b. Plot the graph of field strength versus $1/r^2$ onto the graph paper. Draw a line of best fit through the data and include error bars.

(5)



c. Calculate the gradient of your line of best fit.

(3)

rise = 3.30 – 0.50 = +2.8 ✓ from graph line

run = $(8.0 - 1.3) \times 10^{14} = 6.7 \times 10^{-14}$ ✓ from graph line

gradient = rise / run = 4.18×10^{13} N kg^{-1} m^2 ✓

d. From the gradient of your line of best fit, calculate the mass of Mars. (2)

gradient = $4.18 \times 10^{13} = G \times M$　　M = gradient / G

$M = 4.18 \times 10^{13} / 6.67 \times 10^{-11}$ ✓ $= 6.27 \times 10^{23}$ kg ✓

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Spare Graph Paper

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